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ARCHIMEDES.

The Works of Archimedes. Edited in Modern Notation, with introductory Chapters by T. L. Heath, Sc.D., sometime Fellow of Trinity College, Cambridge. Pp. clxxxvi + 326. (Cambridge: at the University Press, 1897.)

THIS is a companion volume to Dr. T. L. Heath's valuable edition of the "Treatise on Conic Sections" by Apollonius of Perga, and the same patience, learning and skill which have turned the latter book into a delightful guide to early Greek geometry have been here applied to present in a most readable form the extant works of perhaps the greatest mathematical genius that the world has ever seen. The same general plan of editing has been followed in this book as in that of Apollonius, such condensation and modernisation having been introduced as is possible without alteration of the methods employed. We consequently now possess for the first time in English dress a reproduction of Archimedes' work, without addition or essential omission, from which have been removed the thorns and briars that have hitherto beset the path of the English student who would make himself acquainted with the methods invented and employed by the extraordinary genius of Archimedes in order to make those measurements which now-a-days are rendered easy only by the use of the integral calculus. Not the least deterrent of these obstacles have been the Doric dialect of parts of the original and the abbreviations and corruptions in which the text of the accessible editions abounds, so that hitherto the French translation by Peyrard, or that in German by Nizze, has been preferable for the purposes of study.

The volume itself is a handsome example of mathematical printing such as we are wont to expect from the Cambridge University Press, and it is most fitting that it should have been issued by that Press, since Torelli's standard edition (in Greek with a Latin version) was published by and at the expense of the sister University under the editorship of Prof. Abram Robertson, D.D., of Christ Church.

The book will do much to restore Archimedes to his proper place in the estimation of mathematicians; for, in spite of the admirable histories of mathematics that lie to our hand, Archimedes is scarcely known at all except as the discoverer of an important principle in hydrostatics, or the constructor of a spiral, or the inventor of a screw, or the destroyer of ships by the use of a mirror. Indeed it is but little known that in his books on Equilibrium he laid the foundation of theoretical mechanics, and that in his treatises on Floating Bodies he created the science of hydrostatics; that he it was who discovered $3\frac{1}{2}$ as a superior limit of the ratio of the circumference of a circle to its diameter, and $3\frac{1}{9}$ as a still closer lower limit; that to him we owe the quadrature of the conic sections and of the surfaces generated by their revolution, as well as the cubature of the volumes so formed; or that he was the author of a system of representing numbers up to that which would now be expressed by 1 with eighty thousand million millions of

cyphers following it. What else he may have done we do not know for certain, as our information rests on vague accounts by later authors; but eight at least of his published works are lost, and these cannot but have shown the same originality and been marked by the same presentation of new truths as those which have survived, while none of them, or of those that we have, relate to his numerous mechanical inventions, of which in comparison with his mathematical speculations he seems to have thought meanly since, in Plutarch's words, he would not deign to leave behind him any written work on such subjects.

Not the least important part of this book is the introduction of 186 pages which precedes the 326 pages of text. This contains a very valuable discussion of the problems that attracted the attention of the early Greek geometers, and also a most interesting account of the various suggestions that have been put forward as to the mode in which Archimedes obtained certain results which he states without entering into details respecting them.

After a preliminary recital of the stories told of Archimedes by ancient writers, and a short account of the ingenious mechanical inventions they have attributed to him, Dr. Heath discusses the MSS. sources of the text that survive, and enumerates with slight details the principal editions of their text that have been published. From this we learn all that is really needful, though the Latin version with commentary made by Abbot Maurolycus in 1534-49, and published at Palermo in 1685 by Cyllenius Hesperius, also deserves notice. One might further remark that if the titles of the different editions are given at all in anything like fullness, they should be given with absolute exactness, and this is not done in any of the Latin versions save those of Torelli and Heiberg.

Then follows an excellent chapter upon the relation of Archimedes to his predecessors, and this is full of interest not only historically, but also as contrasting Archimedes' methods with those of others, and illustrating his extraordinary facility in the manipulation of proportions, a special instance of which is, in modern notation, the elimination of b, c, d from the conditions $a/b = b/c = c/d > 1$, $3d(a - c) = 5x(a - d)$, $(a - c)(2a + 4b + 6c + 3d) = 5y(a + 2b + 2c + d)$. Next comes a clear account of the Greek system of numerals and the Greek modes of performing arithmetical operations, more especially with reference to the approximations to the square roots of numerics which are not squares; but in this last matter there is plenty of room for conjecture, as not much direct information is available, and Dr. Heath gives a fairly exhaustive account of what has been written on the subject. The fifth chapter treats of the problems known as *μεύεις*, which deal with the straight lines that have to verge towards a given point, and fulfil some other condition too; these have not much to do with Archimedes, but they are exceedingly interesting, and without their discussion a study of Greek geometry would be incomplete. The Greek methods of geometrically solving problems that practically involve cubic equations are next unfolded at length, and a discussion of the classification of problems and loci as plane, solid and (curvi-) linear completes a most interesting chapter. Finally Archimedes' anticipations of the integral calculus

by an admirable extension of the method of exhaustion are considered, and copiously illustrated; and the introduction concludes with a most useful account of the mathematical terms and phraseology employed by Archimedes.

Dr. Heath's general treatment of the text is excellent, but there is one point on which we do not think that he has exercised a sound judgment. This is the renumbering of the propositions in three of the books, which renders reference to them a matter of entire uncertainty. Thus the celebrated Prop. 37 of Book I. on the Sphere and Cylinder, which gives $\frac{2}{3}$ as the ratio of the surfaces and of the volumes of a sphere and the cylinder that just contains it, appears in Dr. Heath's book as a corollary of Prop. 36, which is numbered 34; he further puts Props. 33, 43, 45, 46 as corollaries, and includes Prop. 1 in the introductory letter, thus reducing to 44 the 50 propositions of the MSS. In Book II. he treats Prop. 1 similarly, even though Archimedes himself, in the introductory letter to his work on Spirals (p. 152), mentions this proposition as the first of those which were proved in this book. In the book on Conoids and Spheroids he further puts as a Lemma what appears as Prop. 1 in all the Greek and Latin texts, so that the famed Prop. 12 on certain plane sections of these figures, the characters of which are said by Archimedes to be *φανεραί*, appears as Prop. 11, and Prop. 18 is put as the third part of Prop. 17 (numbered 16)—without any reason, since it is not an extension of, though deducible from, the other parts.

Complete as the book is, there is one addition that would be welcomed by all students, viz. a table of all the writers named, and of the approximate dates at which they flourished; this, if of easy reference, would be of great help when the relation of different geometers to any problem was under consideration. A larger number of references is also desirable in the interests of the student who is stirred to go to the fountain-head by his thirst for first-hand information. Thus the proofs on pp. liv–lviii of the above-cited Prop. 12 on Conoids and Spheroids are so introduced, especially in being contrasted with Zeuthen's, as to appear to be quite modern, while really being contained in Torelli's edition, and practically also in the earlier editions of Maurolycus and Rivaltus.

We feel much inclined to challenge Dr. Heath's rendering of Euclid's definition of a straight line in his note on p. 3, and to maintain the correctness of the ordinary version of it. The special point seems to us to lie in the word *ἐπί*, "on," the use of which seems to be due to the thought in the writer's mind of points marked by letters over the line; thus Aristotle ("Ethics," v. 4, 12) writes *ἴσαι αἱ [i.e. γραμμαί] ἐφ' ὧν* AA, BB, ΓΓ ἀλλήλαις where the sentence can only mean "the lines AA, BB, CC are equal to one another"; the words must therefore mean "the lines are equal over the two ends of which A and A, B and B, C and C are written," so that Euclid's *τὰ ἐφ' αὐτῆς σημεῖα* must mean "its own extreme points."

Such slight blemishes, however, or what we deem to be such, do not detract from the very great value of this work, and Dr. Heath deserves our grateful thanks for the labour he has expended upon it. It is a fitting addition

to his former task; for as, to quote Charles, in Apollonius work we find the origin of the geometry of Forms and Situations as it is now developed, so in the present book we are introduced to the basis of the geometry of Measurement which has demanded a new calculus for its perfection. R. E. B.

A NEW TEXT-BOOK OF EXPERIMENTAL PHYSIOLOGY.

The Essentials of Experimental Physiology. For the use of Students. By T. G. Brodie, M.D., Lecturer on Physiology, St. Thomas's Hospital Medical School. Pp. xiv + 231. (London: Longmans, Green, and Co., 1898.)

THIS volume appears in the same series as two books already well known to the medical student of this country, viz. Schäfer's "Essentials of Histology" and Halliburton's "Essentials of Chemical Physiology." It is bound uniformly with these two volumes, but on opening the books all resemblance disappears. Instead of being divided into a number of "lessons," each of suitable length for one day's work of a practical class, the new volume is divided into "chapters" the length of which bears no proportion to the practical work of a class-meeting. In the other two volumes of the series, each lesson is commenced by a concise and definite description for the student of how to perform a number of experiments; and this description is followed by a few pages of theoretical teaching bearing on those experiments. In the present volume this arrangement is departed from, and the experiments are interpolated in a discursive fashion through the text.

An attempt is made to mark off by heavy type a portion of the book as an elementary course, but it is questionable whether this would not have been much better done by dividing the volume into an elementary and advanced course, as has been done by Prof. Halliburton in the "Essentials of Chemical Physiology." Again, some of the experiments described in this elementary portion are quite beyond the reach of the junior student—for example, the maximum work performed during a muscular twitch; while other simple and important experiments, such as the effect of temperature on muscular contraction, are excluded from this part.

The book commences with a description of various forms of galvanic cells and the chemical changes involved in their action. Such a description is scarcely necessary, for the student is supposed to be already familiar with the commoner forms of galvanic cell before commencing practical work in physiology; but, if inserted, it would have been much better to have described the action of the cells correctly. For example, the student who has just attended a junior course of instruction in a physical laboratory will be somewhat disturbed in his notions of electrolysis by being told that when the Daniell's element is in action—

"the chemical changes in the battery are, solution of zinc and formation of ZnSO_4 at the zinc plate, and decomposition of the CuSO_4 by the hydrogen appearing at the copper plate to form H_2SO_4 and metallic Cu, which latter is deposited on the copper surface."